Exercise 13

For the following exercises, rewrite the quadratic functions in standard form and give the vertex.

$$f(x) = 3x^2 - 5x - 1$$

Solution

Begin by factoring the coefficient of x^2.

$$f(x) = 3\left(x^2 - \frac{5}{3}x - \frac{1}{3}\right)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -\frac{5}{3}$, which means $B = -\frac{5}{6}$ and $B^2 = \frac{25}{36}$. Add and subtract $\frac{25}{36}$ on the right side within the parentheses and use the identity so that x appears in only one place.

$$f(x) = 3\left[\left(x^2 - \frac{5}{3}x + \frac{25}{36}\right) - \frac{1}{3} - \frac{25}{36}\right]$$

$$= 3\left[\left(x + \left(-\frac{5}{6}\right)\right)^2 - \frac{37}{36}\right]$$

$$= 3\left[\left(x - \frac{5}{6}\right)^2 - \frac{37}{36}\right]$$

$$= 3\left(x - \frac{5}{6}\right)^2 - \frac{37}{12}$$

Therefore, the vertex of the parabola is $\left(\frac{5}{6}, -\frac{37}{12}\right)$.